ANALYSIS OF THE METHOD OF TEMPERATURE MEASUREMENT USED TO DETERMINE THE SET OF CHARACTERISTICS OF A THERMALLY PROTECTIVE COATING

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Methods of optimum measurement planning are used to choose an efficient scheme for the placement of thermocouples in a study of characteristics of the thermal state of a thermally protective coating.

Temperatures are commonly measured by means of thermocouples in tests conducted on thermally protective coatings to study their thermal state. The temperature field in the coating is usually determined by sensors in which the thermocouples are placed different distances from the heated surface in accordance with a certain scheme.

To make thermal tests more informative, it is interesting to explore the use of data from temperature measurements in such sensors to determine other heat-transfer characteristics in a coating — such as thermophysical characteristics of the material, characteristics of thermal loading, etc. Methods based on the solution of inverse heat-conduction problems (ICP) [1, 2] can be effectively used to determine these characteristics, the accuracy of such methods depending on the scheme of temperature measurement adopted in the experiment [3, 4].

The optimum temperature measurement schemes may differ for different characteristics, with respect to both the number of measurement points and their location in the specimen [5, 6]. Thus, the problem arises of selecting an efficient thermocouple placement scheme which will ensure that reliable results are obtained and that the entire set of characteristics being studied will be accurately determined.

Here, we propose that this problem be solved by taking an approach in which one first chooses optimum measurement schemes for the determination of individual characteristics. Then, on the basis of comparative analysis of these schemes, a general temperature measurement scheme is chosen. Particular problems involving measurement planning are solved using the methods and algorithms presented in [5-8].

We will examine thermal tests whose goal is to determine the nonsteady temperature field $T(x, \tau)$, $0 \le x \le \delta$, $0 \le \tau \le \tau_m$, and the temperature of the heated surface $T_1(\tau)$ and to refine the thermal conductivity $\lambda(T)$ of a thermally protective coating of glass-fiber plastic with an organosilicon binder on the basis of temperature measurements made in a flat specimen of thickness δ (Fig. 1a) subjected to one-sided heating in a high-enthalpy gas flow. It is assumed that determination of the temperature field in the specimen is the most important goal of the test. We will therefore subsequently analyze this problem in greater detail. We must use ICP methods to determine the relations $T_1(\tau)$ from the readings of thermocouples embedded in the specimen because direct measurement of temperature from thermocouples placed on the surface does not always yield satisfactory results. This may be due either to destruction of the thermocouple or to disturbance of its contact with the material by thermal and mechanical loads.

It is assumed that the loss of mass by the specimen during its failure in the tests is negligible. Thus, the process of heat transfer in the specimen can be readily described by a mathematical model in the form of a boundary-value problem for the heat-conduction equation:

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right),\ 0 < x < \delta,\ 0 < \tau \leqslant \tau_m, \tag{1}$$

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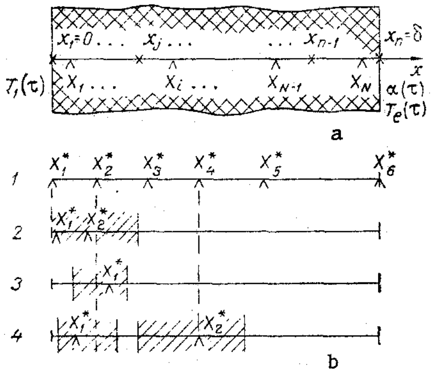

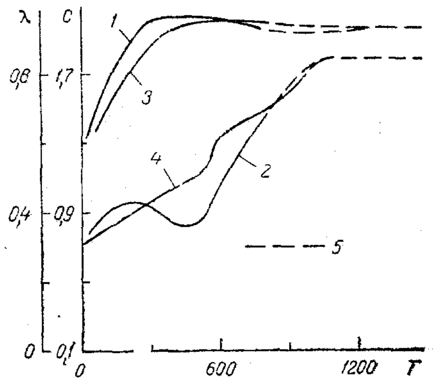

Fig. 1 Fig. 2

Fig. 1. Schemes of location of measurement points in the specimen: design scheme (a) and optimum measurement schemes (b) to determine the following: 1) $T(x, \tau)$; 2) $T_1(\tau)$; 3) $\lambda(T)$ at N = 1; 4) $\lambda(T)$ at N = 2.

Fig. 2. Thermophysical characteristics of the material: 1) C(T); 2) $\lambda(T)$ for variant No. 1; 3) C(T); 4) $\lambda(T)$ for variant No. 2; 5) results of extrapolation. λ, W/(m·deg); C, J/(m^3·deg); T, °C.

$$T(x, 0) = T_0(x), \quad 0 \leqslant x \leqslant \delta, \tag{2}$$

$$T(0, \tau) = T_1(\tau), \quad 0 < \tau \leqslant \tau_m, \tag{3}$$

$$\lambda(T(\delta, \tau)) \frac{\delta T(\delta, \tau)}{\delta x} = \alpha(\tau)[T_e(\tau) - T(\delta, \tau)], \quad 0 < \tau \leqslant \tau_m. \tag{4}$$

Furthermore, it is assumed that the temperature dependences of the thermophysical characteristics (TPC) of the material $\lambda(T)$ and C(T) are determined from the solution of the ICP [9, 10] and correspond to curves 1 and 2 in Fig. 2. The specimen heating regime corresponds to the change in temperature $T_1(\tau)$ on the heated surface (curve 1 in Fig. 3). Heat-transfer conditions with the parameters $\alpha(\tau)$ = 5.81 W/(m²·deg), $T_e(\tau)$ = 15°C are reproduced on the internal surface. The initial temperature distribution $T_0(x)$ = 15°C. The duration of the process τ_m = 175 sec. The thickness of the specimen δ = 0.019 m.

We introduce the measurement scheme or plan:

$$\xi = \{N, \overline{X}\}, \ \xi \in \Xi, \ \overline{X} = \{X_i\}_1^N, \tag{5}$$

$$\Xi = \{(N, \overline{X}) : N \geqslant N_{\min}, \ X_i \in \Omega, \ i = \overline{1, N}\},$$

where X_i are space coordinates giving the location of the measurement points on the x axis (Fig. 1a). The set of possible measurement schemes Ξ is determined by the specific planning problem [5, 7, 9].

With allowance for the results in [7], the problem of optimizing the scheme of measurement of ξ_T^* in the determination of the temperature field $T(x, \tau)$ is formulated in the form of the following extremal problem:

$$\xi_T^* = \operatorname{Arg}\min Z(\xi_T), \ \xi_T = \{N, \overline{X}\}, \ N \geqslant N_{\min}, \ \overline{X} = \{X_i\}_1^N,$$

$$0 \leqslant X_i \leqslant \delta, \ i = \overline{1, N}.$$

The optimum coordinates of the thermocouples X_i^*, $i = \overline{1, N}$, at a fixed number N of thermocouples, are determined from the solution of the extremal problem:

$$\overline{X}^* = \min_{\overline{X}} F(N, \overline{X}), \ \overline{X} = \{X_i\}_1^N, \ 0 < X_i < \delta, \tag{6}$$

$$i = \overline{2, N-1}, \ X_1^* = 0, \ X_N^* = \delta.$$

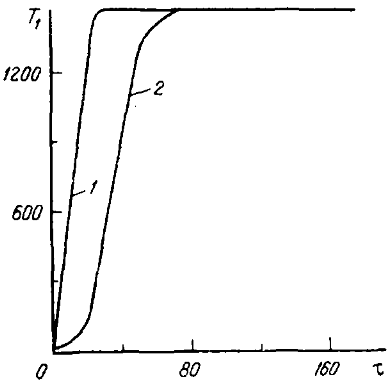

Fig. 3

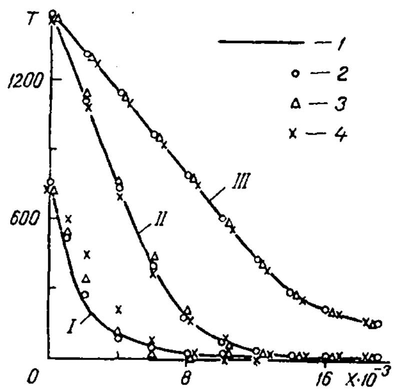

Fig. 4

Fig. 3. Change in specimen surface temperature: 1) regime No. 1; 2) regime No. 2. τ, sec.

Fig. 4. Temperature distribution in the specimen for the moments of time τ = 12.5 (I); 50 (II), and 175 sec (III): 1) exact values; 2) values determined from measurements at points in the optimum scheme; 3) values determined from measurements at points in the scheme with a uniform thermocouple arrangement; 4) values determined in the presence of the "failure" of a thermocouple at the point X_2^*. X, m.

The optimum number of thermocouples N^* is chosen by successively increasing the value of N by unity on the basis of the condition: $Q(N, \bar{X}) \le \varepsilon$, $N = N_{min}, N_{min} + 1, \ldots$, where problem (6) is solved for each fixed value of N. We use the following functions as planning criteria

$$F(N, \bar{X}) = \frac{\sum_{l=0}^{L} \sum_{j=0}^{n} [S(x_j, \bar{X}, \tau_l) - T(x_j, \tau_l)]^2}{\sum_{l=0}^{L} \sum_{j=0}^{n} [T(x_j, \tau_l)]^2}, \tag{7}$$

$$Q(N, \bar{X}) = \max_l \left[\frac{\max_j |S(x_j, \bar{X}, \tau_l) - T(x_j, \tau_l)|}{\max_j |T(x_j, \tau_l)|} \right], \tag{8}$$

$$l = \overline{1, L}, \quad j = \overline{1, n},$$

these criteria characterizing the accuracy of approximation of the relation $T(x, \tau)$ by the splines $S(x, \bar{X})$. Here, $T(x_j, \tau_\ell)$ are values of temperature assigned on a two-dimensional grid: $\Delta_{nL} = (x_j, \tau_\ell)$, $j = \overline{1, n}$, $\ell = \overline{1, L}$, formed by two one-dimensional grids: Δ_n: $0 = x_1 < x_2 < \ldots < x_j < \ldots < x_n = \delta$ (see Fig. 1a) and Δ_L: $0 = \tau_1 < \tau_2 < \ldots < \tau_\ell < \ldots < \tau_L = \tau_m$; $S(x_j, \bar{X}, \tau_\ell)$ are values of the spline $S(x, \bar{X})$ at the nodes of the grid Δ_n constructed on the network of nodes X_i, $i = \overline{1, N}$ for each moment of time τ_ℓ, $\ell = \overline{1, L}$; ε is a quantity characterizing the required accuracy in the determination of $T(x, \tau)$. In solving the planning problem, we assume that the measurements are made continuously over time and without errors. We use cubic interpolational splines as the functions $S(x, \bar{X})$. The minimum allowable number of nodes $N_{min} = 3$ for these splines.

The theoretical values of temperature $T(x_j, \tau_\ell)$, $\ell = \overline{1, L}$, $j = \overline{1, n}$, used in the planning as a priori information on the temperature field, are determined from the solution of the specimen heating problem (1)-(4).

The measurement planning problem being examined was solved using the numerical algorithm proposed in [7] with the following initial data: $\delta = 0.019$ m, $\tau_m = 175$ sec, $N_{min} = 3$, $\varepsilon = 0.1$. The table of values of temperature was assigned on a uniform space-time grid with the number of nodes n = 21, L = 21. As the initial approximation of the measurement scheme in the solution of problem (6), we used a uniform location of the measurement points on the interval [0, δ].

TABLE 1. Coordinates of Points of Optimum Measurement Schemes (X_i^*, m)

Measurement scheme	Characteristic determined	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*
ξ_T^*	$T(x, \tau)$	0,0	0,00265	0,00548	0,00851	0,01212	0,01900
ξ_T^p	$T(x, \tau)$	0,0	0,00380	0,00760	0,01140	0,01520	0,01900
$\xi_{T_1}^*$	$T_1(\tau)$	0,0	0,00200	—	—	—	—
$\xi_{\lambda(N=1)}^*$	$\lambda(T)$	0,00333	—	—	—	—	—
$\xi_{\lambda(N=2)}^*$	$\lambda(T)$	0,00143	0,00855	—	—	—	—

TABLE 2. Values of Characteristics of the Accuracy of Determination of the Temperature Field in the Presence of Thermocouple "Failures"

"Failure" point	x_2^*	x_3^*	x_4^*	x_5^*	No "failure"
$\bar{Q}$	0,131	0,060	0,050	0,077	0,037
F	$0,119 \cdot 10^{-2}$	$0,615 \cdot 10^{-3}$	$0,776 \cdot 10^{-3}$	$0,297 \cdot 10^{-2}$	$0,120 \cdot 10^{-3}$

The results of the calculations showed that the sought optimum scheme ξ_T^* is the measurement scheme with the number of points $N^* = 6$ and the coordinates shown in Table 1. For comparison, the table also gives the coordinates for the scheme ξ_T^p with a uniform thermocouple placement.

To study the effect of the parameters of the measurement scheme on the accuracy of the temperature field determination, we mathematically modeled the experiments and then analyzed their results. As the measurement results, we used the theoretical temperatures $T(X_i, \tau)$, $i = \overline{1, N}$ obtained from the solution of problem (1)-(4). We examined optimum and uniform thermocouple placement schemes and we evaluated the effect of errors in the thermocouple coordinates and "failures" of individual thermocouples on the accuracy of the temperature field determination.

Figure 4 shows results of determination of characteristic temperature profiles T(x) by means of spline-approximation for τ = 12.5, 50, and 175 sec. The results show that the optimum measurement scheme provides for the prescribed accuracy of temperature field determination throughout the ranges of the variables x and τ. The accuracy characteristics (7) and (8) have the following values: $F = 0.12 \cdot 10^{-3}$, $Q = 0.068$. Comparison of the results for the schemes ξ_T^* and ξ_T^p shows that use of the optimum measurement points increases the accuracy of the temperature field determination severalfold (for ξ_T^p, $F = 0.367 \cdot 10^{-3}$, $Q = 0.149$).

Table 2 shows data characterizing the effect of "failures" of thermocouples located at interior points (X_i^*, $i = \overline{2, 5}$) in the optimum scheme on the accuracy of determination of the temperature field. Here, for greater convenience in analyzing the results, we have replaced the characteristic Q by $\bar{Q}$. The latter quantity is defined by the relation

$$\bar{Q}(N, \bar{X}) = \frac{\max\limits_{j,l} |S(x_j, \bar{X}, \tau_l) - T(x_j, \tau_l)|}{\max\limits_{j,l} |T(x_j, \tau_l)|}, \quad l = \overline{1, L}, \; j = \overline{1, n}.$$

The results of the calculations show that failures of thermocouples located at points X_2^* and X_5^* have the greatest effect on the accuracy of the temperature field determination. Here, a failure at point X_2^* has a greater effect on $\bar{Q}$, while a failure at point X_5^* has a greater effect on F.

Figure 4 shows the results of determination of the temperature field in the presence of a failure at point X_2^*. Here, it should be noted that the failures of thermocouples located on the heated (X_1^*) or internal (X_6^*) surfaces of the specimen make it impossible to determine the relation $T(x, \tau)$ with the requisite accuracy on the interval $[X_1^*, X_2^*]$ or

$[X_5^*, X_6^*]$. This is because of the loss of information on the temperature field at the corresponding boundaries of the specimen, although such information can be found from the solution of an inverse boundary-value problem of heat conduction [1] using measurements of temperature at interior points of the specimen.

In analyzing the effect of errors of thermocouple placement on the quality of the measurements, we used the theoretical values of temperature $T(\tilde{X}_i, \tau)$, $i = \overline{1, 6}$, at points whose coordinates were determined by the relation $\tilde{X}_i = X_i^* - \Delta x$, $i = \overline{2, 5}$, $\tilde{X}_1 = X_1^*$, $\tilde{X}_6 = X_6^*$, where ΔX are the deviations of the coordinates from the optimum values. We took ΔX = 0.0002, 0.0005, and 0.001 m in the calculations. The results of the calculations, depicted in Table 3 in the form of the relation $F(\Delta X)$, show that the deviation of the coordinates from the optimum values within the range $\Delta X = \pm 0.0005$ m does not lead to a significant increase in the error of the temperature field. It should be noted that analysis of the effect of the error of thermocouple placement is particularly important for the interior points of the measurement scheme, since it is difficult to check their location in the specimen.

To evaluate the effect of indeterminacies in the thermophysical characteristics of the material and the heating regime on measurement planning, we selected optimum measurement schemes with N = 6 for variant No. 2 of TPC values $\lambda(T)$ and $C(T)$ (see curves 3 and 4 in Fig. 2) and heating regime No. 2, which corresponds to the relation $T_1(\tau)$ (curve 2 in Fig. 3). Comparing the coordinates of the measurement schemes shown in Table 4, we find that the TPC deviation seen in the calculations (Fig. 2) leads to a substantial change in the optimum coordinates. A change in the heating regime (Fig. 3) has considerably less effect on the coordinates of the measurement points.

We used the results in [5, 6] to analyze problems involving the selection of optimum temperature measurement schemes $\xi_{T_1}^*$ and ξ_λ^* in the determination of the relations $T_1(\tau)$ and $\lambda(T)$ by the ICP method. The authors of [5, 6] made detailed examinations of the formulation of similar problems, as well as methods and algorithms for solving them.

The planning problem is formulated in the form of the following extremal problem: $\xi^* = \operatorname{Arg\,max} \psi[M(\xi)]$, $\xi \in \Xi$, where the set of possible measurement schemes (5) is determined by the relations: $\Xi = \{(N, \overline{X}): N \geq 1,\ 0 \leq X_i \leq \delta,\ i = \overline{1, N}\}$ in the determination of $T_1(\tau)$ and $\Xi = \{(N, \overline{X}): N \geq 1,\ 0 < X_i < \delta,\ i = \overline{1, N}\}$ in the determination of $\lambda(T)$.

We used the determinant of the normalized Fisher information matrix $\det M$ [11] as the planning criterion $\psi[M(\xi)]$. The calculations were performed using the application package described briefly in [12] and the following initial data: $\delta = 0.019$ m, $\tau_m = 175$ sec, $T_0(x) = 15°C$, $T_e(\tau) = 15°C$, $\alpha(\tau) = 5.81$ W/(m²·deg). The thermophysical characteristics of the specimen material were assumed to correspond to variant No. 1 (see curves 1 and 2 in Fig. 2), while the change in the temperature of the heated surface corresponded to regime No. 1 (Fig. 3). We took m = 6 as the number of parameters in the representation of the relations $T_1(\tau)$ and $\lambda(T)$ by cubic B-splines, this number ensuring that the relations would be described with an error no greater than 10%. The temperature fields and the sensitivity functions were calculated on a uniform space-time grid with the number of nodes $n_x \times n_\tau = 41 \times 41$.

TABLE 3. Values of the Characteristic F in the Presence of Errors in the Coordinates of the Thermocouples

ΔX, m	−0,001	−0,0005	−0,0002	0	0,0002	0,0005	0,001
$F\cdot 10^3$	0,226	0,133	0,122	0,120	0,125	0,156	0,190

TABLE 4. Coordinates of Measurement Points in the Determination of the Temperature Field for Different TPC and Heating Regimes (X_i^*, m)

Heating regime	TPC variant	X_1^*	X_2^*	X_3^*	X_4^*	X_5^*	X_6^*
1	1	0,0	0,00265	0,00548	0,00851	0,01212	0,01900
1	2	0,0	0,00163	0,00580	0,00930	0,01510	0,01900
2	2	0,0	0,00180	0,00552	0,00935	0,01504	0,01900

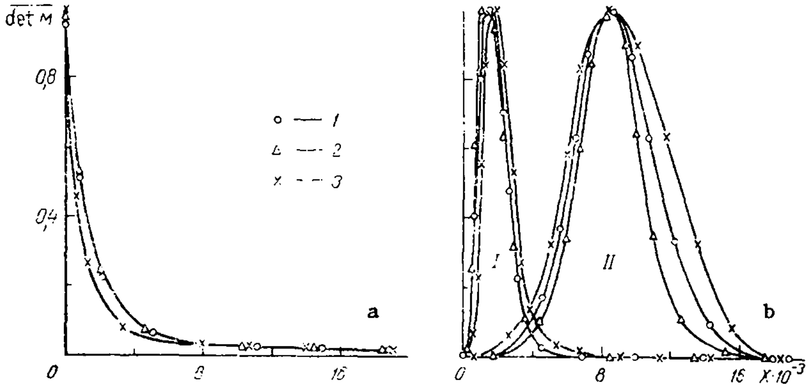

Fig. 5. Dependence of the criterion $\overline{\det M}$ on thermocouple location: a) in the determination of surface temperature ($X_1^* = 0$, $0 \le X_2 \le 0.019$ m): 1) regime No. 1, TPC variant No. 2; 2) regime No. 1, TPC variant No. 1; 3) regime No. 2, TPC variant No. 1; b) in the determination of thermal conductivity [I) $0 < X_1 < 0.019$ m, $X_2 = X_2^*$; II) $X_1 = X_1^*$, $0 < X_2 < 0.019$ m].

Table 1 shows the optimum coordinates of the measurement points to determine $T_1(\tau)$ with $N = 2$ and $\lambda(T)$ with $N = 1, 2$. Figure 5 shows the dimensionless criterion $\overline{\det M}(\overline{X}) = \det M/(\det M)_{max}$ as a function of thermocouple location for two variants of TPC and two heating regimes.

The results show that location of the thermocouples on the heated surface is optimum for the determination of $T_1(\tau)$. The sensitivity of the system decreases as the measurement points are located farther from the surface, this decrease leading to a reduction in the accuracy of the determination of the characteristic. The character of relation $\det M(\overline{X})$ shows that there is a preferred region for thermocouple placement ($0 \le x \le 0.005$ m) around the heated surface. This was confirmed by the results of mathematical modeling in which a study was made of the dependence of the accuracy of solution of the inverse boundary-value problem of heat conduction on the location of temperature sensors [3, 5].

In the case where several measurement points are used ($N > 1$) and the first point is on the specimen surface, the position of the other points has almost no effect on the sensitivity of the system at $X > 0.005$ m.

Comparison of relations $\det M(\overline{X})$ for heating regimes Nos. 1 and 2 shows that the change in heating regime examined here does not lead to a change in the coordinates of the optimum measurement scheme. There is negligible change in the character of the relations $\overline{\det M}(\overline{X})$. Indeterminacy in the TPC of the material, corresponding to the difference between variants Nos. 1 and 2 (Fig. 2), also has almost no effect on the planning results.

Analyzing the effect of possible thermocouple failures, we should note that with the use of several measurement points, failure of the first thermocouple (X_1), located on the heated surface, leads to the largest loss of accuracy in the determination of $T_1(\tau)$ in the case where the remaining thermocouples are located at the distance $X_i > 0.005$ m. The failure of thermocouples installed at points X_i, $i = 2, 3, \ldots$, has less effect on the accuracy of $T_1(\tau)$. If all of the measurement points are located near the heated surface ($0 \le X_i \le 0.005$ m), then the failure of one of the thermocouples will not significantly reduce the accuracy of determination of the characteristic in question.

It can be concluded on the basis of the above analysis that the optimum scheme for determining $T_1(\tau)$ is the scheme with two points located as closely as possible to the heated surface. If design or process limitations prevent the use of this scheme, it would be best to locate the first point (X_1) as close as possible to the heated surface and to place the second point (X_2) as close as possible to the first point with allowance for the given limitations (an example of such a restriction would be that there be a minimum allowable distance between thermocouples). If we assume that in our case the restriction has the form $(X_2 - X_1) \ge 10\emptyset$, then for diameter of the thermoelectrode $\emptyset = 0.2$ mm, $X_2 \cong 0.002$ m.

Analysis of the optimum measurement schemes for the determination of $\lambda(T)$ shows that, with $N = 1$, there is a rather narrow region near the heated surface ($0.0015\ m \le x \le 0.005\ m$) in which placement of the thermocouples will ensure a high degree of accuracy for the solution of the ICP. If we successively increase the number of measurement points, we first ($N = 2$) see a sharp increase in the optimum values of the planning criterion $\overline{\det M}$ (by a factor of more than 60) and a substantial change in the optimum thermocouple coordinates. A further increase in N has almost no effect on the value of the criterion. This indicates that an increase in N would be inexpedient from the viewpoint of increasing the accuracy of solution of the ICP and that we can restrict ourselves to an optimum scheme containing just two measurement points ($N^* = 2$). The results obtained here agree well with the data obtained in [4, 6, 9] from parametric analysis of the accuracy of an ICP solution.

Indeterminacy in the TPC of the material and the heating regime has almost no effect on the planning results within the difference of the curves in Figs. 2 and 3. The coordinates of the optimum measurement schemes obtained for heating regimes Nos. 1 and 2 and TPC variants Nos. 1 and 2 coincide to within the size of a step ($\Delta x = 0.000475$ m) on the space coordinate grid. The character of the relations $\overline{\det M}(\overline{X})$ changes little.

Regarding the effect of possible thermocouple failures on the determination of $\lambda(T)$, it should be noted that the chosen measurement scheme ξ_λ^* is most sensitive to a loss of information at the point with the coordinate $X_1^* = 0.00143$ m. When the second thermocouple ($X_2^* = 0.00855$ m) fails, we obtain a measurement scheme with one point whose coordinate is displaced relative to the optimum value for $N = 1$.

Deviations of the coordinates of the thermocouples from their optimum values within the region of their preferred location have little effect on the accuracy of the determination of $T_1(\tau)$ and $\lambda(T)$. However, unchecked errors in the determination of the thermocouple coordinates may produce unsatisfactory results in the solution of the ICP [1].

A comparative anlaysis of optimum measurement schemes ξ_T^*, $\xi_{T_1}^*$, ξ_λ^* is made in Table 1 and Fig. 1b, where the regions where the thermocouples can be placed are indicated. The analysis shows that measurement scheme ξ_T^* can be used to reliably determine the entire set of heat-transfer characteristics in the coating. Here, $T_1(\tau)$ should be determined from the readings of thermocouples positioned at the points X_1^* and X_2^*, while $\lambda(T)$ should be determined from the readings of thermocouples installed at points X_2^* and X_4^* in the optimum scheme ξ_T^*.

NOTATION

T, temperature; x, coordinate; τ, time; τ_m, duration of the process; δ, specimen thickness; $T_1(\tau)$, temperature of the heated surface; $\alpha(\tau)$, heat-transfer coefficient on the inside surface of the specimen; $T_e(\tau)$, ambient temperature; $T_0(x)$, initial temperature distribution; $\lambda(T)$, thermal conductivity; C(T), volumetric heat capacity; ξ, measurement scheme (plan); N, number of measurement points; N_{min}, minimum permissible number of measurement points; $\overline{X} = \{X_i\}_1^N$, vector of coordinates of the measurement points; X_i^*, $i = \overline{1, N^*}$, optimum coordinates of the measurement points; Ξ, set of possible measurement schemes; Ω, region of location of measurement points; ξ_T^*, $\xi_{T_1}^*$, ξ_λ^*, optimum measurement schemes to determine the relations $T(x, \tau)$, $T_1(\tau)$, and $\lambda(T)$, respectively; ξ_T^P, scheme with a uniform measurement-point location; $Z(\xi)$, functional; F, Q, $\overline{Q}$, accuracy characteristics; ε, required accuracy of the determination of $T(x, \tau)$; $S(x, \overline{X})$, cubic spline; ΔX, deviation of the coordinates; $\psi[M(\xi)]$, $\det M$, $\overline{\det M}$, planning criteria; m, number of parameters in the representation of the sought relations $\lambda(T)$ and $T_1(\tau)$; Δ_{nL}, Δ_n, Δ_L, networks of nodes; n, L, number of space and time nodes in the networks, respectively. Indices: i, number of measurement point; ℓ, j, numbers of nodes of space–time network.

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EFFECT OF DIFFERENT FACTORS ON THE ACCURACY OF THE SOLUTION OF A PARAMETRIZED INVERSE PROBLEM OF HEAT CONDUCTION

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Results are presented for a mathematical simulation of the effect of the error in approximation of the estimated function, the error in temperature measurements, and the error in specifying measurements on the accuracy of the solution of the parametrized boundary-value inverse problem.

Methods based on solving boundary-value inverse problems of heat conduction are widely used at present in the experimental investigation of processes of heat interaction of a solid body with the surrounding medium. In these problems we seek thermal boundary conditions and restore the temperature field in the body based on results of thermal measurements at separate internal points.

In many cases heat transfer in systems being investigated can be described with accuracy sufficient for practical purposes by the one-dimensional nonlinear heat equation

$$C(T)\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right),\quad T=T(x,\tau),\quad x\in(0,b),\quad \tau\in(\tau_{\min},\ \tau_{\max}]. \tag{1}$$

As boundary conditions for (1) we specify the initial temperature distribution

$$T(x,0)=T_0(x),\quad x\in[0,b] \tag{2}$$

and boundary conditions of the second kind

$$-\lambda(T)\frac{\partial T}{\partial x}(0,\tau)=q(\tau),\quad \tau\in(\tau_{\min},\tau_{\max}], \tag{3}$$

$$\lambda(T)\frac{\partial T}{\partial x}(b,\tau)=u(\tau),\quad \tau\in(\tau_{\min},\tau_{\max}], \tag{4}$$

where $u(\tau)$ is an unknown function. We assume that we have data on temperature measurements for the inner surface of the sample:

$$T_{\exp}(0,\tau)=f(\tau). \tag{5}$$

One of the methods for determining the unknown boundary condition $u(\tau)$ for a nonlinear heat equation is to solve the inverse problem by means of minimization of the root-mean-square dispersion of calculated temperature values at the points of fixing of thermal sensors $T_{\exp}$ from the experimentally measured values f. Two cases are possible: 1) we seek a solution in a finite dimensional space of parameters; 2) we solve the optimization problem in a functional space. The first approach is realized when the unknown function $u(\tau)$ is approximated by a certain system of basis functions, for example, by piecewise-constant functions [1], V-splines [2], etc.

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